



The Hong Kong University of Science and Technology

Department of Mathematics

Seminar on Applied Mathematics

Why Phase-Field?

by

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Abstract

The phase-field method has gained considerable attention in the recent years in the communities of materials science, physics and applied mathematics. Although its basics have been established a long time ago, there is still considerable confusion in the literature about the interpretation of phase-field models with respect to the “diffuseness” of the interface between phases. This confusion is caused by the historical development of today’s phase-field models as the merger between two fundamentally different routes.

The first route, for which the brand “phase-field” has been coined by Jim Langer in 1978, is the traveling wave solution for the Korteweg-de Vries equation, which is commonly named a “soliton” in physics literature. This solution is applied to emulate the motion of a sharp interface in a moving boundary problem, e.g., the Stefan problem of solidification, by a diffuse interface. Here, the diffuseness has no physical meaning but is used as a free parameter for convenience in a numerical solution. Such models are characterized by the existence of a so-called “thin interface limit”, which means that a variation of the interface thickness, in certain limits, does not affect the solution. These models are commonly called “mesoscopic phase-field models” [1] since they are agnostic of an intrinsic length scale but the scale emerges from the solution of the coupled problem.

The second route is van der Waals theory of a diffuse surface of a liquid droplet in equilibrium with its vapour. They start out from a variational derivation of the governing equations of motion from a generalized functional of the system under consideration, which includes a penalty against the formation of interfaces. This is the famous “gradient contribution”, as first used in Ginzburg-Landau or Cahn-Hilliard type models. Since within these models, by construction, the interface energy scales with the interface width, they can only be applied at the nano-scale and are called “physical order parameter models” or “microscopic phase-field models. [2].

The presentation shortly reviews the historical development of today’s phase-field models and tries to answer the question “Why phase-field?” by highlighting theoretical and numerical features of the phase-field method alongside selected examples from current research.

[1] Steinbach I. 2013. *Phase-Field Model for Microstructure Evolution at the Mesoscopic Scale. Annu. Rev. Mater. Res. 43:89–107*

[2] Chen L-Q. 2002. *Phase-field models for microstructure evolution. Annu. Rev. Mater. Res. 32:113–140*

Date: Thursday, 29 March 2018
Time: 3:30p.m. – 4:30p.m.
Venue: Room 2504, Academic Building
(near Lifts 25 & 26), HKUST

All are welcome!